Considering Stochastic Mortality in Pricing Variable Annuities – Applications of the Lee Carter Model

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Chao Liang, FSA

Mortality assumptions are crucial in many actuarial areas, such as the pricing of long-dated guarantees in life insurance, pensions and annuities. Due to its direct impact on various calculations in the insurance industry, appropriate assumptions can greatly help reduce mispricing risk. For example, in the past years, people lived longer than expected and the improvement of mortality has caused considerable losses to pension writers.

The stochastic projection of mortality improvements would help actuaries better quantify the mortality/longevity risk associated with certain products. Given a clearer picture about the gains/losses from mortality improvements at different confidence levels, actuaries can make more informed decisions in pricing and risk management.

Lee Carter Model

First introduced in 1992, Lee Carter model has become a widely used model in stochastic mortality forecasting.[1] The following represents the stochastic equation that drives mortality rates levels in future time steps:

$$\ln(m_{x,t}) = a_x + b_x k_t + \epsilon_{x,t}$$

$m_{x,t}$ is the central death rate for age x in year t, which is driven by a single time-varying parameter k_t that captures mortality trend, on top of a parameter a_x that describes the general shape across age of the mortality schedule, b_x captures the sensitivity in each age to the change in overall mortality level. They are age-specific constants. $\epsilon_{x,t}$ is an error term that captures age specific historical behaviors not captured in the model.

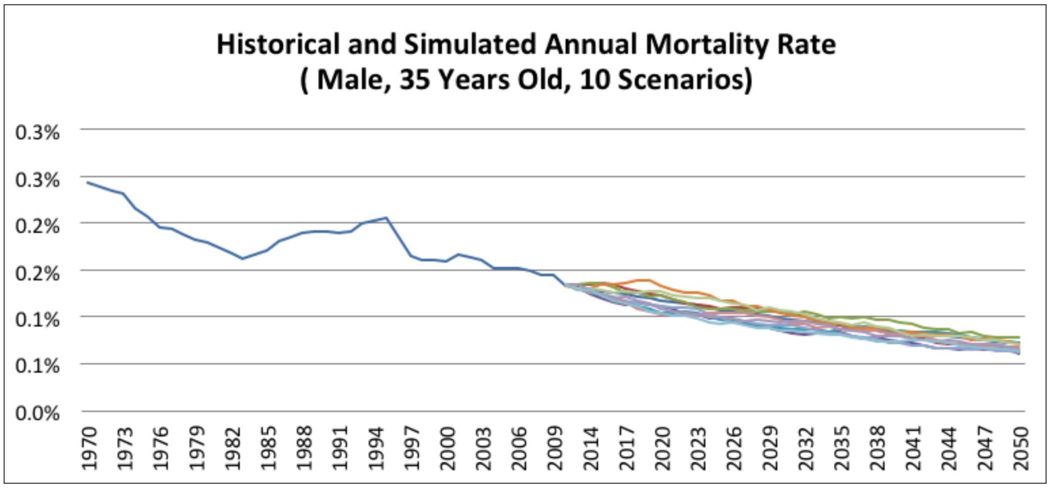
The model can be estimated by fitting to historical mortality in past decades. Simulating the mortality trend into the future, we can generate the level and age distribution of mortality at future time steps.

Chao Liang, FSA
Insurance Product Specialist
cliang@numerix.com
Tel: +1 212 302 2220 ext. 113

Numerix LLC
Corporate Headquarters
150 East 42nd Street
15th Floor
New York, NY 10017
Tel: +1 212 302 2220

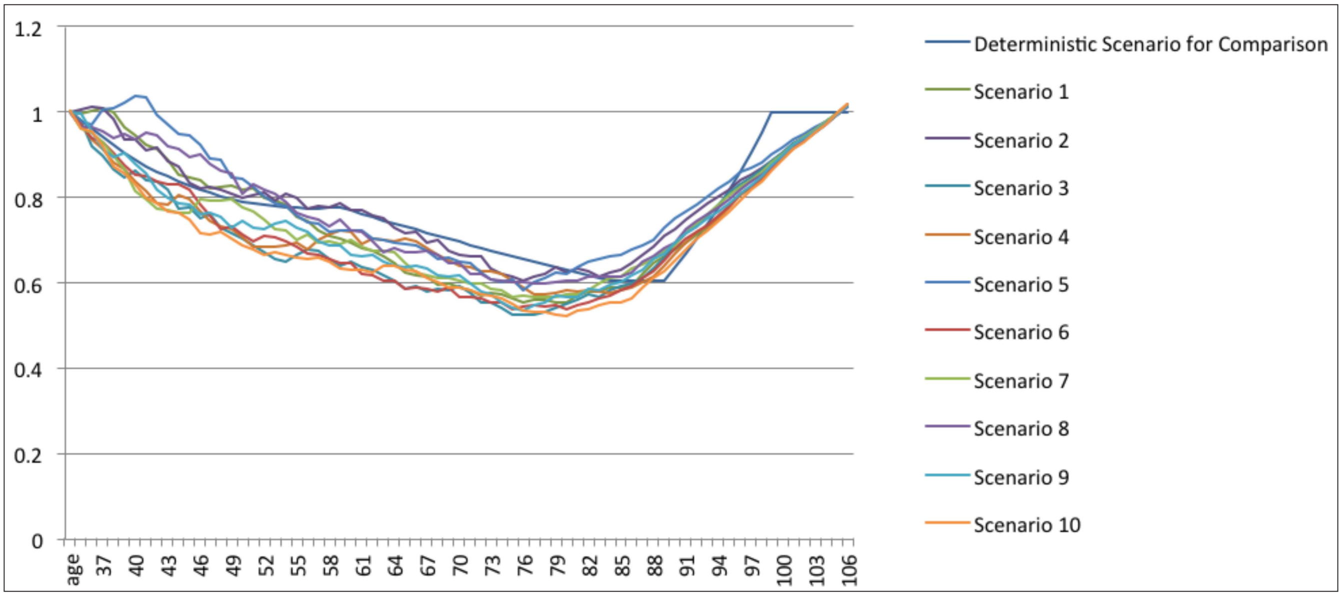
[1] *Ronald D. Lee and Lawrence R. Carter (1992), "Modeling and Forecasting U.S. Mortality", Journal of the American Statistical Association.*

The graph below shows one example of simulated mortality rate for a male, with fixed age equal to 35.



Deterministic vs. Stochastic Mortality Rates

Without a stochastic model, actuaries may assume a constant mortality improvement rate each year or may roughly apply adjustment factors on mortality rates to set deterministic assumptions. What is a more realistic stochastic mortality assumption? Lee Carter model assumes that a single component drives the overall mortality improvement, while taking into account sensitivities in each age group with respect to the overall improvement. The following graph illustrates the simulated mortality rates as percentage of original mortality rate for the cohort of 35 years old at start date. A deterministic assumption is also shown for comparison purposes.



As it is shown, the mortality improvement in different scenarios could have so much discrepancy. Take an annuity product for example, if annual payment is made until death, a change in mortality scenario could cause large moves in benefit payment cash flows, as well as in the present value of these cashflows.

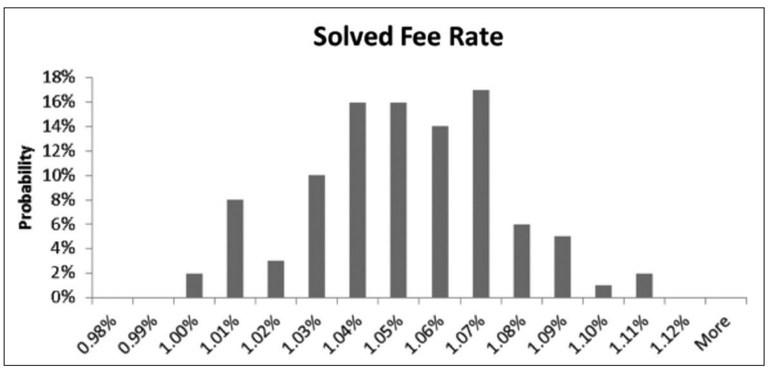
Implication in Pricing

How does a stochastic mortality analysis help in pricing? Let's take the example of a typical GMWB product, and solve for the rider fee what the insurer needs to charge in order to cover the exact present value of the guarantee. For that, we use stochastic scenarios on rates, equities as well as simulating profit and loss for stochastic mortality scenarios.

Simplified case - GMWB product

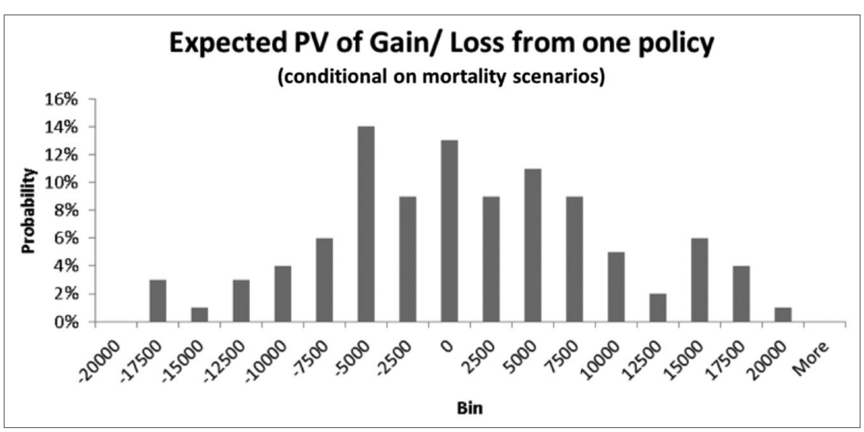
- *35 years old when policy is issued*
- *Annual ratchet High Water Mark for Guaranteed Base, starts from 55 years old*
- *Withdrawal Eligibility starts from 65 years old*
- *Guaranteed withdrawal amount = 5% of Guaranteed Base for life*
- *GMWB rider fee is paid out of account value annually*

Under the deterministic mortality assumption, an annual fee of around 1.05% of the rolling account value will cover the cost of the lifetime withdrawal guarantee. With the stochastic mortality scenarios generated by Lee Carter model, we can draw a distribution of the breakeven point for the rider fee.



Potential Gain/ Loss based on Deterministic Mortality Rates

Assume the insurer charges 1.05% annual fee rate for this GMWB rider based on the deterministic mortality scenario, and the initial account value is $2,000,000. In this case, what is the chance that mortality risk would cause loss to the company? How severe could the loss be due to unexpected mortality scenario? Using stochastic mortality scenarios generated by Lee Carter model, we could see that the expected present value of gain or loss per policy conditional on different mortality scenarios is distributed as below.



Clearly, in about half of the scenarios the company would bear a loss due to bad longevity scenarios. If we consider the actual loss amount in the future (as opposed to a present value), and compare the distribution (with regards to market risk) of accumulated gain/loss in the deterministic mortality scenario to the same distribution under an adverse mortality scenario, the average loss amount in the 30% worst scenarios (CTE(70)) is increased by more than $500,000 (based on a $2m initial Account Value). Therefore, the mortality/longevity risk may have material impact on overall risk profile of the product and should not be ignored compared to other risks. We could increase the fee to add some cushion for adverse change of mortality rates. By moving the fee rate to different levels we could recalculate the gain/loss distribution, and this would help provide more information about the product's risk profile.

CONCLUSION

As a major risk in long term insurance products, mortality risk has material impact on liabilities calculation. Incorporating the numerical method based on Lee Carter model to the hybrid framework makes it possible to quantify all major risks associated with insurance products, and is especially beneficial for those bearing long term mortality/longevity risks, such as GMWB, annuities, pensions and other long term insurance products.

AUTHOR BIOGRAPHY

Chao Liang, FSA, Insurance Product Specialist, Numerix LLC

Ms. Liang is an insurance product specialist in the Client Solutions Group, focusing on developing insurance product solutions and business analysis. Prior to joining in Numerix, she was an actuarial consultant working on insurance liability valuation, capital requirement calculation and embedded value calculation.

Ms. Liang holds a Master of Financial Engineering degree from UCLA. She is a Fellow of the Society of Actuaries.

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Numerix LLC Corporate Headquarters

100 Park Avenue
15th Floor
New York, NY 10017
Tel: +1.212.302.2220
Fax: +1.212.302.6934
www.numerix.com