Alternative VaR Models

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Abstract

We describe a variety of VaR models in terms of their key attributes and differences, e.g., parametric vs. nonparametric, historical sampling vs. Monte Carlo simulation. We start with the simpler, well known models and then describe randomized historical simulation and filtered historical simulation, highlighting the features and benefits of these alternative methods. Filtered historical simulation has some unique attributes that could make it a better alternative for managing risk.

Introduction

Value at Risk (VaR) is a measure of market risk that expresses it as the $P^{th}$ percentile loss in value of a portfolio, i.e., the maximum amount that could be lost with $P\%$ probability. Many of the techniques for calculating VaR simulate a number of possible scenarios, each representing a different set of market conditions, and then value a portfolio in each of those scenarios to create a distribution of portfolio gains and losses from which the $P^{th}$ percentile loss is determined. In other words, the distribution of portfolio gains and losses is derived from the distribution of scenarios, so to create a realistic set of gains and losses, one must create a realistic set of scenarios.

$P$ is typically 95% or 99%, so for a given distribution, the VaR loss is in the tail of the distribution, which corresponds to relatively rare scenarios. That has implications for the techniques used to calculate VaR.

There is a variety of models for generating the scenarios, and the choice of model basically reflects risk management’s view of how realistically it can generate a set of scenarios that represents possible future market conditions. All of the models incorporate some information about past behaviour of market data, such as the volatilities of each risk factor and the correlations between different risk factors. It is usually assumed that these statistical measures will stay nearly the same over the relatively short time horizons into the future for which VaR is calculated. It is also assumed that the mean return is zero. The different models have different ways of ensuring that these statistics are reproduced in generated scenarios.

Another choice is how to weight historical data. One could take the view that historical changes which occurred in the recent past are no more likely than those which occurred in the distant past (equal weights), or one could take the view that historical changes in the recent past are more likely than those in the distant past (unequal weights).

The time horizon for VaR is fairly short, usually one to ten days. This time horizon affects the relative size of the market data changes (longer times imply larger changes). One might expect that VaR
would be calculated by applying N day market data changes to current market data, then valuing the portfolio N days forward of the current date, but that is not how VaR is typically calculated. Valuing a portfolio at a forward time requires portfolio aging, which is difficult. So, the market data changes are treated as instantaneous changes and the portfolio is valued as of the current date. This is equivalent to assuming that for the short time horizons used in VaR, the portfolio will not change significantly. This might not be a good assumption, e.g., for options near their maturity dates.

Some VaR models are based on sampling from a set of historical changes (non-parametric) while others calculate statistical measures of historical changes and use those to parameterize a process that generates possible changes (parametric) and others combine elements of both (semiparametric).

**Variance/ Covariance**

This simple parametric model assumes the distribution of portfolio returns is a multivariate normal distribution. The historical data are used to calculate the covariances of returns from which VaR can be calculated without any simulation of scenarios. VaR follows from standard analytic results of normal distributions.

**Advantages**

This model is very simple and easy to understand.

**Disadvantage**

The actual distributions of portfolio returns are not normal, they have fat tails and other differences that significantly impact VaR.

**Historical Simulation**

One of the simplest models for VaR uses historical data to determine a set of day over day changes to market data that have actually occurred for some period, then applies each one of those changes to current market data to generate scenarios. The idea is that if a certain set of daily changes occurred over a span of time in the recent past and affected the value of a portfolio, then that is a pretty good representation of the possible one day changes in value of the portfolio that might occur from the current day to the next day.

The changes in market data used are exactly the changes that actually occurred in the recent past. The time period over which these changes are calculated is typically one year (for regulatory purposes) or perhaps several years, and the end date in the period is typically the current date.

A year has approximately 250-260 business days, so that is about how many scenarios can be generated and used to create the gain/loss distribution. In statistical terms, that is a very small sample size, especially when considering the tail of the distribution which drives the VaR measures. For example, the 99th percentile loss is one where only two scenarios will have a larger loss than the VaR value.

Equal weight is given to day over day changes in the recent past (near the end date) and to those in the distant past (near the start date). This is equivalent to the view that a change that occurred any
time in the past is as likely as a change that occurred recently, and also that the volatility of past changes is the same as the volatility of recent changes.

**Advantages**

The historical simulation model is a simple method of generating scenarios. It is relatively easy to calculate day over day changes in market data and apply them to current market data.

All historical mean values, volatilities, and correlations are embedded in the historical data itself and the model includes them without any extra effort.

The small number of scenarios means relatively small computational requirements.

**Disadvantages**

The number of scenarios is statistically small, so the resulting VaR number has a fair amount of day to day instability associated with it.

If a large day over day change occurs, it will tend to drive VaR until it drops out of the time period over which VaR is calculated, e.g., one or more years. In this case, the VaR calculation is more like a stress scenario for that one particular stress than a simulation of many possible scenarios. If it is driving VaR, then even if very little is changing in the market or in the portfolio, the day it drops off can see a big change in VaR, which is not very realistic.

Historical changes are assumed to apply regardless of the current values of market data. For example, if the historical data contains a day over day interest rate change at a time when rates were near zero, but current rates are in the double digits, then it might not be realistic to apply that change to current data. For this reason, the historical changes are usually calculated and applied as relative changes rather than absolute changes, but that is about as much as the model can do to handle very different market environments. Also, if current volatility is higher than historical values, then this model could underestimate VaR in the current environment because it effectively assumes that the average volatility over the entire historical period is the current volatility.

Historical trends will be reproduced in the simulation, which might not be realistic. For example, if three years of interest rate returns are used, and for the first two of the three years the rates were dropping and then bottomed out near zero in the third year and that is the current situation, then in the simulation two thirds of the scenarios will be trying to push rates down even lower.

The one day VaR is often scaled up to D days by simply multiplying the one day VaR by the square root of D. This is equivalent to the approximation that the distribution of portfolio returns is a normal distribution¹, which it is not (as mentioned above). Scaling this way also neglects portfolio aging effects.

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¹The distribution of a sum of normally distributed variates is also a normal distribution, and it follows from this property that the $P^{th}$ percentile of the distribution scales by the square root of D. This property is not shared by other distributions.
Monte Carlo Simulation

Monte Carlo VaR is a parametric model. Historical data is used to calculate various statistical measures of the distribution of market data changes, then that measures are used directly or indirectly to calculate parameters that drive a Monte Carlo simulation of scenarios. For example, the covariance of the historical market data changes is used to generate sets of market data shifts that have the same covariance.

Monte Carlo simulation can generate arbitrarily large sets of scenarios, including many scenarios which did not actually occur in the historical data set used to calculate the parameters, so the coverage of possible scenarios is good.

The calculation of the parameters is a separate step from the simulation in Monte Carlo VaR, so choices like how much history to use for the calculation of the statistical measures in order to get appropriate values for realistic simulation can be made independently of choices like how many scenarios to generate in the simulation to get acceptable bounds on the simulation statistical error.

Advantages

The ability to generate a large number of scenarios means there can be good coverage of possible scenarios and also that the variance of the simulation can be kept within desirable limits. The variance is a measure of how far the result of doing the simulation with a finite number of scenarios is from the result that the simulation would converge to if there was an infinite number of scenarios. The variance decreases as the number of scenarios increases, so you can achieve a certain variance by running the Monte Carlo simulation with enough scenarios.

The number of scenarios in the simulation is completely independent of the number of historical scenarios used to calculate the parameters of the Monte Carlo simulation. Choosing the number of scenarios is not possible in historical simulation as described above because the number of historical scenarios is the same as the number of simulated scenarios, but it is possible in Monte Carlo.

There are sampling techniques that can be used in Monte Carlo simulation to reduce the variance - antithetic sampling, stratified sampling, etc. - so these are alternatives to increasing the number of scenarios.

The parameter data is relatively small compared to the historical data from which it is derived.

The parameters themselves are information that can make it easier to see intuitively what could be the driving factors of VaR. For example, the volatility and correlation of the risk factors can be obtained from the covariance matrix, so you can see that risk factor A has twice the volatility of risk factor B and risk factor C is highly correlated with risk factor D. This information is not explicit in a non-parametric model like historical simulation.

A single large change in the historical data will factor into the parameters used for the Monte Carlo simulation, but its effect will be muted by the averaging effect of being combined with all the other historical data to derive the parameters. Therefore, it will not have the tendency to drive the VaR number in the way that it would for a historical simulation.

Disadvantages

There are many ways to parameterize a Monte Carlo simulation based on historical data, so many that choices often must be made to reduce the number. For example, there might be dozens of
interest rates at different tenors captured for a single interest rate curve, and it would be prohibitively expensive to simulate each and every rate. One way to handle this is to pick a few parameters that capture the bulk of the changes of the entire curve and simulate those. Each of these choices introduces the possibility of failing to capture significant changes and therefore generate an unrealistic VaR value.

Calibration and back testing is required in order to ensure that the parameters in a Monte Carlo simulation will generate realistic VaR values. This involves using the parameters in multiple simulations for historical dates and showing that the actual losses over that set of historical dates do not exceed the predicted VaR values from the simulations by more than the expected percentage (e.g., 1% if VaR is calculated at the 99th percentile).

A common assumption is that the distribution of each risk factor return can be simulated as a normal distribution with some appropriate mean, volatility and correlation with other risk factor returns.

Historical data is not always consistent with the assumptions inherent in the derivation of the parameters. For example, a common technique is to calculate the historical correlations between risk factors and then generate random samples using a Cholesky decomposition of the correlation matrix. However, this technique can fail when two risk factors are very highly correlated, or when the historical data used to calculate the correlations has gaps due to holidays in certain regions, i.e., where there is no data for that date because it is not a business date. Additional steps have to be taken to handle these numerical issues, and how they are handled imply views that have to be justified from a risk perspective. For example, two highly correlated risk factors could be treated as one independent risk factor and as a second that is a spread to the first. That would solve the numerical issue but would mean a change from modeling the second as an independent risk factor to modeling the spread. Gaps in the data could be handled by assuming that the value on a holiday is the same value as the prior date, or it could be linearly interpolated from the values on surrounding dates.

The methods to condense a large set of parameters into a few (as mentioned above) might have to be fairly sophisticated. For example, one might use principal component analysis (PCA) to determine a small set of factors that capture the bulk of the changes in the historical data. This complexity means additional sources of modeling error and can make it more difficult to understand intuitively the meaning of the factors affecting the simulation.

Recall that VaR is a measure of the tail of the portfolio gain/loss distribution. That means that in order to calculate a reasonably accurate VaR, you need to generate a reasonable number of scenarios in the tail of the simulated distribution, which in turn means you need to generate a lot of scenarios in the rest of the distribution. For 99% VaR, that means that for every generated scenario that leads to a loss above the 99% level, there will have to be 99 others below it. So, while it is possible to generate as many scenarios as required to reach a particular level of accuracy, it can be very expensive.

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2 Techniques such as Principal Component Analysis can be used to determine the most appropriate parameters.

3 This is not a rare occurrence when multiple markets are involved. I once did an empirical analysis of a couple dozen markets with different holidays and found that there were only about sixty business days per year that were not a holiday in any of the markets.
Randomised Historical Simulation

RiskMetrics published a paper in 1997 about how to combine T historical return vectors with random Gaussians for 1 day VaR.[RMM 97]

Even though they refer to it as a Monte Carlo model, it is fundamentally a historical sampling model rather than a parametric model. The authors show that if historical returns are multiplied by uncorrelated, normally distributed random variates, then the generated scenarios will have the same volatilities and correlations as the historical returns. However, the introduction of the random variates means that the number of generated scenarios can be larger than the number of historical returns, unlike a pure historical simulation as described above. This leads to an improvement in the variance of the simulation without the need to explicitly calculate or decompose a correlation matrix.

This model has nearly the simplicity of simple historical simulation but can generate a large number of varying scenarios like the Monte Carlo simulation described above, so it combines the advantages of both and eliminates some of the disadvantages of each.

The original model described in the paper creates each simulated return vector as a linear combination of all of the historical returns. This would require quite a few random variates as well as a fair amount of multiplying of numbers (with 260 days of history and 1000 returns per date, then to generate 5000 scenarios would require 260×5000 random variates and 260×5000×1000 multiplications). However, as they briefly mention, it is not really necessary to combine all historical returns for every scenario to achieve the benefits of this model. In fact, for each scenario, you can pick just one historical date and multiply its returns by one normally distributed random variate. The volatilities and correlations of the generated scenarios will still work out the same. For the same example, that means 5000 random variates and 5000×1000 multiplications.

Advantages

Randomized historical simulation reproduces historical volatilities and correlations automatically, like the simpler historical simulation described above.

This model can generate a large number of different scenarios very easily. This is good for reducing the variance in the simulation for more stable day to day VaR (like Monte Carlo). The large number of scenarios also avoids the excessive impact of one large historical change.

Because it is a historical sampling technique combined with just independent, normally distributed random variates, it avoids numerical issues related to parameterization and calibration.

The mean of the randomized historical returns will be zero even if there is a historical trend (i.e., nonzero mean). The normally distributed random variates have zero mean which makes the simulated returns have zero mean so there is no tendency to bias the simulated returns in the same direction as the historical returns. However, see the section on nonzero means below.

Disadvantages

A historical date with a large gain can result in a simulated scenario with a large loss because each historical return is multiplied by a normally distributed random variate, and negative variates are as likely as positive variates. This is mitigated somewhat by the fact that the large number of scenarios will tend to smooth out the effects of large historical returns, whether negative or positive.
The distribution of the generated returns will be a normal distribution even if the distribution of the historical returns is not normal.

**Weighted Historical Simulation**

All of the VaR models described so far have applied equal weights to historical returns. A risk manager could take the view that market data changes in a VaR calculation are more likely to be similar to changes in the recent past than in the distant past. A VaR calculation can take this into account by weighting the more recent historical returns more than the more distant historical returns.

For the historical sampling techniques that involve randomly choosing a historical date for each scenario, equal weighting corresponds to choosing a random number from a uniform distribution. Unequal weighting corresponds to choosing a random number from some other distribution, e.g., exponential, which is straightforward.

For Monte Carlo, the historical data used to calculate the parameters can be weighted. For example, when calculating means and covariances of historical returns, varying weights can be assigned and standard formulae for weighted means and weighted covariances can be used.

**Nonzero Historical Means**

For the various models based on historical returns, there can be historical trends in the data, i.e. nonzero means. If desired, these can be removed in the simulation. There are a couple of ways of doing this.

The historical returns can be mirrored by adding the negative of each historical return to the sample. This will double the number of samples (which is good) as well as making the mean zero, but it will also change the form of the resulting distribution of returns. For example, if the distribution of the historical returns happens to be a normal distribution with a nonzero mean, then the distribution of the historical returns plus their mirrors will not be a normal distribution (though it will be close if the mean is small relative to the variance).

The covariance of the RiskMetrics randomized historical returns described above will not match the covariance of the historical returns if there is a nonzero mean. If the mean of risk factor $n$ is $\mu_n$, and the mean of risk factor $m$ is $\mu_m$, and their covariance is $\sigma_{nm}$, then the covariance of the simulated returns will be $\sigma_{nm} + \mu_n\mu_m$, which could be larger or smaller than the covariance of the historical returns depending on whether the two means have the same sign or not. The diagonal terms of the covariance (where $n=m$) are the volatilities of the risk factors, so this implies that the volatility of the simulated returns of risk factor $n$ will be $\sigma_n + \mu_n^2$; it will always be larger than the volatility of the historical returns of the same risk factor.\(^4\)

The historical returns can be normalized to have a zero mean by just subtracting the mean value from each return. This will preserve the sample variance. This can be done before doing simple historical simulation, mirrored historical simulation or the randomized historical simulation to preserve the covariances in all cases.

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\(^4\) The RiskMetrics paper only considers the case where the means of the historical means are zero. This result in the case of nonzero means was derived at TFG.
Filtered Historical Simulation

Filtered Historical Simulation (FHS) attempts to combine the benefits of parametric and nonparametric models into a semiparametric model [FHS]. It uses sampling of historical returns to get the proper correlations and distribution of simulated returns, but replaces the historical volatilities with volatilities obtained from current market data. The issue it addresses is that unfiltered historical simulation tends to underestimate VaR in high volatility periods and overestimate VaR in low volatility periods because it effectively uses a constant volatility that is the average over the whole period. By replacing the constant, average volatility with one taken from current market data, VaR will be higher in high volatility periods and lower in low volatility periods.

FHS is also used for N day VaR by combining N one day returns. At each step, the forward volatility obtained from current market data is used, so it effectively has a time dependent parameterization of the volatility. This is not possible with nonparametric models.

FHS can be viewed as a weighting model where the recent data for volatility (the current market conditions) are weighted 100% while the past data for volatility are weighted 0%.

The focus of FHS seems to be on the volatility of the returns but little mention is made of the mean. If the historical returns have a nonzero mean, then FHS will reproduce that trend if the returns are not normalized. If they are normalized, then the simulated returns will have a mean of zero, which also might not be realistic.

The idea of scaling the volatility as described above can be generalized to the idea that current market data can be used for many different types of risk factors to allow including current market data information about future expectations into a simulation. The general process is to normalize historical data and then scale it to current values. At TFG we have shown that this technique can be used to make the mean of simulated interest rate returns match the current curve while preserving the historical correlations.

Advantages

FHS provides the benefits of historical sampling, i.e., the automatic inclusion of correlations and shape of the historical distribution without any explicit assumptions about either.

FHS allows parameterizing important parts of the simulation without requiring parameterizing everything (as in the pure Monte Carlo simulations).

As a parametric model, FHS could be extended to incorporate additional information beyond the historical returns that could be important for VaR. As described above, it captures current market information about implied volatilities, but other data could be included as well, like autocorrelation (correlation of day over day changes).

Disadvantages

FHS assumes that the volatility of the returns is independent of the correlations of those same returns, which is not necessarily true. This issue carries over from unfiltered historical simulation, where all of the historical statistical properties, including both correlations and volatility, are assumed to have the same values in the current environment, so it is no worse in that regard.

FHS requires a slight increase in complexity of the simulation compared to unfiltered historical simulation.
FHS requires current market data for each risk factor from which it can determine the volatility that it will use to scale the normalized historical volatilities.

Summary

We described several VaR models: Variance/Covariance, Historical Simulation, Monte Carlo, Randomized Historical Simulation and Filtered Historical Simulation. These fall into the categories of parametric, semiparametric and nonparametric. Advantages and disadvantages of each model were highlighted. The nonparametric, historical sampling models are generally easier to implement than the parametric Monte Carlo model and avoid the calibration process, but a major issue is that they tend to reproduce historical trends that may not be desirable and the simpler models do not generate many scenarios, which can lead to a fair amount of daily variance in VaR. Filtered historical simulation attempts to combine the benefits of the nonparametric models with the benefits of the parametric models while avoiding the major issues of both. FHS could be a better tool for managing risk than the standard models (often implemented because they are required for regulatory purposes), particularly if portfolio aging could be included to allow for longer term simulations.

Bibliography
